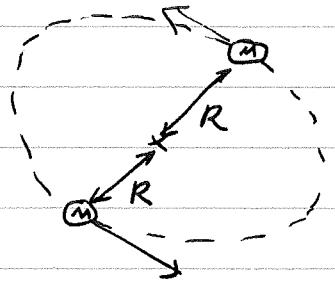
B5 Gravitational Radiation II

Due end HT6

Merton College

1. (Some Newtonian Physics)
State and prove the (Newtonian) virial theorem.

Consider two masses, mass M, seperated by a distance 2R and orbiting their centre of mass.



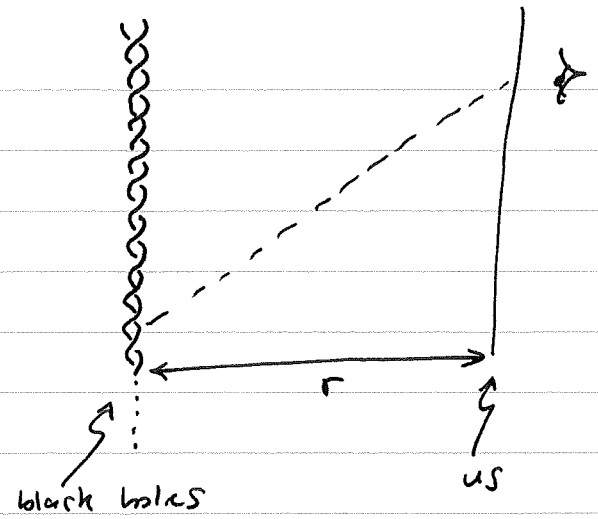
What is the total energy of the system? What is the angular velocity of the masses?

2. (Radiation From Binary System)
Consider two black holes orbiting each other. We approximate their motion using Newtonian physics (Q1). So, in (t, x, y, z) coordinates,

$$T_{00} = M\,\delta(z)\big[\,\delta(x - R\cos\omega t)\,\delta(y - R\sin\omega t) + \delta(x + R\cos\omega t)\,\delta(y + R\sin\omega t)\,\big].$$

We observe the binary from a distance r. We experience a metric $g = \eta + h$, where h is small. What is h_{ij} to leading order in $1/r$?

[Hint: use HT5, Q5.]



What is the frequency of the waves we observe? Show that the amplitude of the waves we observe satisfies

$$\text{ampl} \propto \frac{G}{r} E_{tot}$$

where E_{tot} is the total (Newtonian) energy of the binary.

3. (Energy Lost By Binary system)
In units where $c=1$, show that the binary system radiates with power

$$P = -\frac{2G^4M^5}{5R^5}.$$

Restore c to this equation.

[Hint: you may use $P = -\frac{1}{5} G \ddot{J}_{ij} \ddot{J}^{ij}$, where $J_{ij} = I_{ij} - \frac{1}{3}\delta_{ij} I_{kk}$ and dots denote derivatives with respect to t.]

4. (Life Span of Binary)

Estimate the life span of the binary by finding a differential equation for $R(t)$. If $R = R_0$ at $t = 0$, you should find that $R \to 0$ at

$$t_0 = \frac{5}{32} \frac{R_0^4}{G^3 M^3}.$$

(Restore factor of c.)

[Hint: use the Newtonian expression for E_{tot} and write $\dot{E}_{tot} = P$.]

5. (Detection of The Radiation)

Suppose our interferometer is sensitive to changes in distance up to one part in 10^k.

A binary system, mass M, is located some distance r away. How close do the black holes need to be for us to detect their radiation?

Using Q4, show that we would detect radiation from such a system for a duration

$$T \propto \frac{G^5 M^5}{r^4} \times 10^{4k}.$$

Find the numerical prefactor and restore factors of c.