Week 1 ~~Ø~~Work

(You must read the 'lecture notes to understand these questions!)

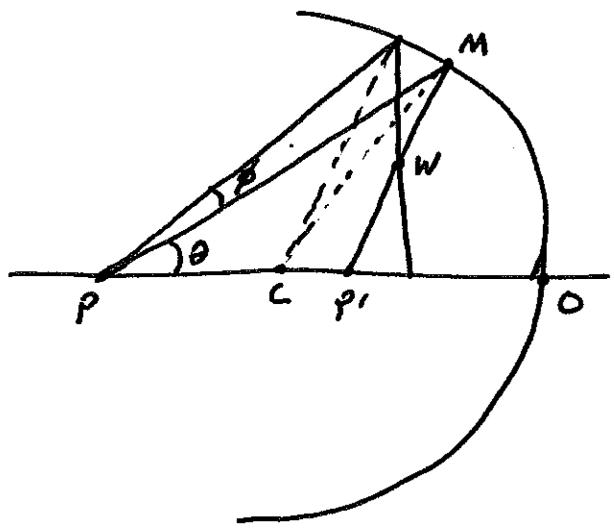
I. All prep problems

& problems from the 1st lecture.

Especially:

$$\frac{s-r}{r-s'} = \frac{\sin\theta'}{\sin\theta}$$

II. Consider two rays reflecting from a spherical reflector



Find the length WM, given the angles θ, ϕ, the distances $\begin{cases} OC = r \\ OP = s \\ OP' = s' \end{cases}$

III. Maxwell's problem: show that the paths of rays moving in medium with

$n(r) = \frac{1}{1+r^2}$ are circles.

IV. Show that Einstein's prediction of the deflected light is:

$$\Delta \phi = \frac{4GM}{b}$$

G: Newton's constant

M: mass of $\odot$

b: distance of closest approach.

(See Lecture notes to get started.)

V. Let T be the time between the CMB surface and today, and t_0 the time today. For some size of the universe function $a(t)$, — with $a(t_0) = 1$ and

$a(T)$ = scale at CMB surface, —

show that we can see a spherical region of the CMB with radius

$$\int_{t_0}^{t_0 - T} \frac{dt}{a(t)} = R.$$

What is R if $a(t) = t^{1/3} / t_0^{1/3}$?

Optics: Week 1 Exercises

i. Make sure you can derive the eqns

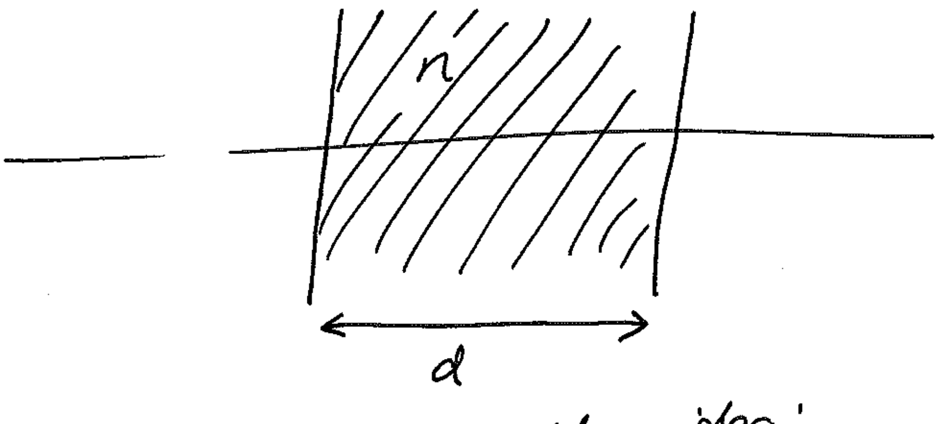
$$\frac{\sin\theta'}{\sin\theta} = \frac{s-r}{r-s'} \quad \text{for a spherical reflector}$$

and

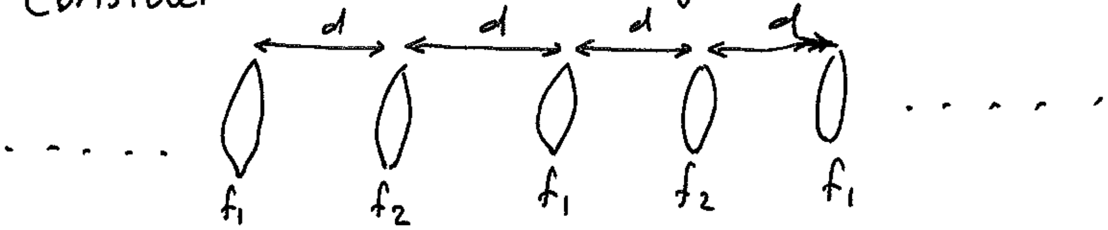
$$\frac{\sin\theta'}{\sin\theta} = \frac{n}{n'} \cdot \frac{s+r}{s'-r} \quad \text{for a spherical refractor}$$

that appear in the notes.

ii. What is the Ray Matrix associated to a flat block of material with refractive index n and shape:



iii. Consider Erik's waveguide idea:



Show that this arrangement is stable for paraxial rays if $\boxed{\left(1-\frac{d}{2f_1}\right)\left(1-\frac{d}{2f_2}\right) < 1}$ using the same method we used for the $f_1 = f_2$ case.

Work: Week 2

I. Q's I & IV from Week 1.

Continue...

II. Recall the formula

$$\frac{1}{f} = (n-1)\left(\frac{1}{r} - \frac{1}{r'}\right)$$

& what it means. Given that f & n are functions of the wavelength λ of light passing through the lens, show that

$$\frac{1}{f}\frac{df}{d\lambda} + \frac{1}{n-1}\frac{dn}{d\lambda} = 0. \qquad (1)$$

For two lenses of foci f_1 & f_2 separated by d, show that the whole system has focal length

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \qquad (2)$$

using ray matrices. Find $d(\lambda_0)$ such that

$$d = d(\lambda_0) \Rightarrow \left.\frac{df}{d\lambda}\right|_{\lambda=\lambda_0} = 0$$

in terms of $\left.\frac{dn}{d\lambda}\right|_{\lambda=\lambda_0}$, $f_1(\lambda_0)$, $f_2(\lambda_0)$, & $n(\lambda_0)$.

The total distance is

$$\mu = \sqrt{x^2+y^2} + \sqrt{(x-p)^2+(y-q)^2}.$$

Suppose the ray had instead reflected at $(\delta x + x, y)$ and ~~the~~ travelled to $(p+\delta p, q)$, where $(p+\delta p, q)$ is slightly off the caustic curve. Then it travels distance

$$\mu' \simeq \mu + A\,\delta p\,\delta x + B\,\delta p\,(\delta x)^2 + C(\delta x)^3 + \cdots.$$

Find

$$A,\ B,\ \text{and}\ C.$$

By defining

$$C^{1/3} z = \delta x + \frac{B\,\delta p}{3C},$$

write

$$\mu' = \text{constant} + m z + z^3$$

for some m. Consider the integral

$$\int_{-\infty}^{\infty} dz\ e^{i\omega(z^3+mz)}$$

and how it might be interpreted.

VI. Consider the gate

$$(a, b, c) \xmapsto{\text{CAND}} (a, b, c \oplus ab).$$

Write out its matrix, M, in the basis

$$\left\{ \begin{matrix} 000, & 100, & 010, & 110, \\ 001, & 101, & 011, & 111 \end{matrix} \right\}.$$

Show that $\boxed{M^T M = \text{Identity.}}$

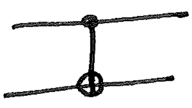
VII. Show that

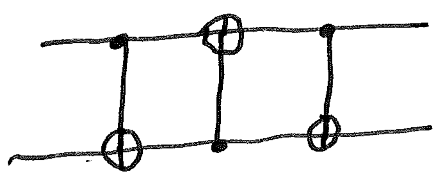
$$H = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \frac{1}{\sqrt{2}}$$

$$D_\theta = \begin{bmatrix} e^{i\theta} & 0 \\ 0 & 1 \end{bmatrix}$$

$$R_\phi = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}$$

are unitary and find θ_0, ϕ_0 such that $D_{\theta_0} R_{\phi_0} = e^{i\psi} H$ for some ψ.

VIII. Recall that  is an abbreviation for $(a, b) \longmapsto (a, a \oplus b)$. What is the effect of



?